

Is mathematical reasoning just reasoning about mathematics?

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PhiMSAMP-3: Is Mathematics Special?

Is informal mathematical reasoning just the application to mathematics of methods of reasoning common to other discourse, or is it distinctively mathematical?

Four possible answers:

- 0 There is no such thing as 'informal mathematical reasoning': only formalized reasoning, in which the inferential steps are those admissible within a formal system, can count as mathematical.
- 1 Informal mathematical reasoning is possible, but must employ exclusively mathematical inferential steps, albeit ones that may be characterized informally.
- 2 Informal mathematical reasoning is possible, and may employ both exclusively mathematical inferential steps, and inferential steps of more general application.
- 3 Informal mathematical reasoning is possible, and may be understood purely in terms of inferential steps of general application. No exclusively mathematical inferential steps are required for informal mathematics; that is, such steps may ultimately be reduced to instances of general steps.

0: No such thing

It cannot be denied that a complex sequence of interlocked blind guesses and cruel rejections may look much like directed thought, just as Darwinian evolution simulates orthogenesis or design. But **we must not be hoodwinked into thinking that it is reasoning, or anything else that we know, that drives us forward to what is unknown.** What reasoning does is pull us back. Our guesses are not random, of course, but informed; which means only that they are guesses informed by earlier guesses.

David Miller, 'Do we reason when we think we reason, or do we think?' *Learning for Democracy*, Vol. 1, No. 3, 2005

1: Exclusively mathematical steps mandatory

For false diagrams of geometrical figures are not contentious (for the resulting fallacies conform to the subject of the art)—any more than is any false diagram that may be offered in proof of a truth—e.g. Hippocrates' figure or the squaring of the circle by means of the lunules. But Bryson's method of squaring the circle, even if the circle is thereby squared, is still sophistical because it does not conform to the subject in hand. So, then, any merely apparent reasoning about these things is a contentious argument, and any reasoning that merely appears to conform to the subject in hand, even though it be genuine reasoning, is a contentious argument: for it is **merely apparent in its conformity to the subject matter, so that it is deceptive and plays foul**

Aristotle, *De Sophisticis Elenchis*, 171b.

2: Exclusively mathematical steps not mandatory

[H]ow much **observation, divination, induction, experimental trial, and verification, causation**, too (if that means, as I suppose it must, mounting from phenomena to their reasons or causes of being) have to do with the work of the mathematician

J. J. Sylvester, 'The study that knows nothing of observation,'
British Association for the Advancement of Science, Exeter,
1869.

3: Exclusively mathematical steps not necessary

[M]athematical reasoning is **already in accord with principles and techniques from informal logic**—even if this is unnoticed by the practitioners.

Ian Dove, 'Towards a theory of mathematical argument,'
Foundations of Science, forthcoming.